IE6511 Advanced Topics in Systems Engineering:

MetaHeuristic and Surrogate Optimization

Assigned: Thursday February 1, 2018

Due: Wednesday February 14, 2018

Remember you can work in a team of 2 people (or maybe 3 if you give good reasons to Prof.) NOTE THAT HOMEWORKS WILL BE ASSIGNED EVERY WEEK BUT YOU HAVE TWO WEEKS TO COMPLETE EACH ASSIGNMENT.

**IMPORTANT:** In the interest of being able to answer everyone’s questions on the HW promptly, please post your questions on the IVLE forum. Please title and state your questions *clearly* and *concisely* so that other members of the class may also benefit from your questions. The grader will try to address the questions that have not been adequately answered by course-mates.

**Question 1. Markov Chain Configuration Graph with Random Walk**

Consider the problem given in the figure below **where you are using random walk**.

Let N(Si) = Si plus all nodes connected by an edge; therefore N(S1) ={ S1, S2, S3}. Your objective is to minimize the cost, which is given inside the circles for each Sj.

1. draw the configuration graph and give the probabilities (using the same convention for locating probabilities near the head of the arrow in the corresponding direction and
2. find the transition probability matrix Θ **=P A**.
3. Also find the convergence probability vector π. Check your answer by seeing if π = π Θ. You can solve this by making a good guess or by solving the equation π = π Θ for π. (This is small enough that you can do it by hand if you wish.) You are not being asked to compute **P**M or ΘM for large M. For each case explain your intuition for the reason behind the answer , i.e. why some πi are zero or why some πi  equal each other.

*(In this assignment you are asked to compute the answer to this problem with Random Walk. In later problems below you will be asked to do similar analysis for greedy search and Simulated annealing.)*

**Question 2. Theory-Configuration Graph and transition probability matrix-Greedy**

Consider the problem given in the figure below. In this problem you are asked to solve it for **Minimizing** with Greedy Search. You can assume there is a 50% chance the search will move along an arrow to a better solution. If the solution doesn’t move, it stays put (e.g. it moves on the arrow that goes from circle j to circle j).

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**Question 3. Configuration SA Transition and convergence Theory**

We will use the theory of Markov chain to analyze the Metropolis algorithm as defined below. Metropolis is the name for simulated annealing algorithm running at a fixed temperature. Please note that this version of Metropolis algorithm is different from Homework 2 in that it does not keep the best cost it has found so far.

**Algorithm** Metropolis (*CurS, CurCost, T, M*);

/\* *running Metropolis algorithm at temperature T for M steps \*/*

**Begin**

**Repeat**

*/\* a step begins \*/*

NewS = Neighbor(CurS);

NewCost = Cost(NewS);

∆Cost = (NewCost – CurCost);

**If (∆Cost < 0) Then**

CurS = NewS;

CurCost = Cost(CurS);

**Else**

**If (RANDOM < e-∆Cost/T) Then**

CurS = NewS;

CurCost = Cost(CurS);

**EndIf**

**EndIf**

M = M – 1;

\*/ *a step ends \*/*

**Until (M = 0)**

**End (\* of Metropolis \*)**

The function we are trying to minimize using the algorithm is cost = 2(x1 – x­2)2 + x1. The domains of x1 and x2 are both {0,1}. Neighbors of a state are defined as states obtained by changing the value of one input variable.

Let State1 = (0,1); State 2 = (1,1); State 3 = (0,0); State 4 = (1,0). We obtain a state transition diagram as shown below . Each arrow represents a transition with non-zero probability.



1. Calculate the cost for each state. What is the global minimum? For each transition in the diagram, calculate the transition probability as a function of temperature T. (Some probabilities are already given in the diagram as examples.) Explain why there are no self loops for states 1 and 4
2. Give the transition matrix **P** for Metropolis algorithm running at temperature T = 1. Reminder: the (i,j) entry of **P** is the transition probability from state i to state j
3. Compute **P5**, **P10**, **P1000** using MATLAB or python (or other). If the algorithm starts at the state 1, give the probability distribution for where the algorithm will be after 5, 10 and 1000 steps. What will the probability distribution after,1000 iterations be if the algorithm starts at a random state?
4. Repeat parts b) and c) for T =10.0
5. Repeat parts b) and c) for T = 0.2
6. Compare your answers for the various values of T. Are they consistent with what you expected to see for SA? Explain. To find the global minimum, why do we need to set an appropriate temperature?

**Question 4. GA schema**

Consider a binary string of length 11 and consider the following schema’s.

A B C D

1\*\*\*\*\*\*\*\*\*1, 11\*\*\*\*\*\*\*\*\*, \*\*\*\*\*111\*\*\* and \*\*1\*\*\*1\*\*0\*

1. Which of these 4 schema are in H=[1\*1\*\*\*\*\*\*0\*]?. What is the order and defining length of each of these 4 schema?
2. Under crossover with uniform crossover site selection, calculate a lower limit on the probability of these schemata’s surviving crossover in one generation.

*:*

1. Calculate the probability of surviving mutation in one generation if the probability of mutation is 0.9 at each single bit position.
2. Now calculate the probability of these schemata’s surviving both crossover and mutation in one generation.